

# PRACTICAL # 10

## Statement:

Following the table give the consumption expenditures (C) and disposable income (Y) for some families.

Regress C on Y and test for heteroscedasticity test (omit 4 values).  
Correct it for heteroscedasticity if it is found by assuming that error variance is proportion to  $Y^2$ .

C	Y	C	Y
106	120	150	180
108	122	159	190
114	130	169	200
117	134	172	204
123	140	175	207
130	150	179	209
133	155	182	210
138	160	185	213
142	166	188	216
144	170	191	220

## Solution:

Goldfeld-Quandt Test for Heteroscedasticity:

First of all,

Arrange the data w.r.t independent variable i.e. "Y"

C	Y
106	120
108	122
114	130
117	134
123	140
130	150
133	155
138	160
142	160
144	170
150	180
159	190
169	200
172	204
175	207
179	209
182	210
185	213
188	216
191	220

$$\hat{C}_1 = a_1 + b_1 Y_1$$

Omit 4 central values.

$$\hat{C}_2 = a_2 + b_2 Y_2$$



Now, regress  $c_1$  on  $y_1$  and  $c_2$  on  $y_2$ , we have

$c_1$	$y_1$	$c_2$	$y_2$	$\hat{c}_1 = 11.86 + 0.787y_1$	$e_1^2$	$\hat{c}_2 = -70.34 + 1.193y_2$	$e_2^2$
106	120	169	200	106.3	0.09	168.26	0.5476
108	122	172	204	107.87	0.0159	173.032	1.0650
114	130	175	207	114.17	0.0289	176.611	2.5953
117	134	179	209	117.32	0.1011	178.997	0.00009
123	140	182	210	122.04	0.9216	180.19	3.2761
130	150	185	213	129.91	0.0081	183.769	1.5154
133	155	188	216	133.84	0.7140	187.348	0.4251
138	160	191	220	137.78	0.0484	192.12	1.2544
969	1111	1441	1679	—	1.928	—	10.68

From the data:

$$\Sigma c_1 = 969, \quad \Sigma c_2 = 1441, \quad \Sigma y_1 = 1111, \quad \Sigma y_2 = 1679$$

$$n_1 = 8, \quad n_2 = 8, \quad \Sigma c_1 y_1 = 135809, \quad \Sigma c_2 y_2 = 302777$$

$$\Sigma c_1^2 = 118347, \quad \Sigma c_2^2 = 259985$$

$$\hat{c}_1 = a_1 + b_1 y_1$$

$$b_1 = \frac{n_1 \Sigma c_1 y_1 - (\Sigma c_1)(\Sigma y_1)}{n_1 \Sigma y_1^2 - (\Sigma y_1)^2} = 0.787$$

$$a_1 = \bar{c}_1 - b_1 \bar{y}_1 = 11.86$$

$$\hat{c}_1 = 11.86 + 0.787 y_1$$

and;

$$\sum_{i=1}^n e_1^2 = \Sigma (c_1 - \hat{c}_1)^2$$

$$\hat{C}_2 = a_2 + b_2 Y_2$$

$$b_2 = \frac{n_2 \sum Y_2 C_2 - (\sum Y_2)(\sum C_2)}{n_2 \sum Y_2^2 - (\sum Y_2)^2} = 1.193$$

$$a_2 = \bar{C}_2 - b_2 \bar{Y}_2 = -70.34$$

$$\hat{C}_2 = -70.34 + 1.193 Y_2$$

and,

$$\sum e_2^2 = \sum_{i=1}^{n_2} (C_{2i} - \hat{C}_{2i})^2$$

Testing Procedure:

Step 1:

Null and Alternative hypothesis are:

$H_0$ : There is no heteroscedasticity

$H_1$ : There is heteroscedasticity

Step 2:

Level of significance:

$$\alpha = 0.05$$

Step 3:

Test Statistic:

$$F = \frac{\sum e_2^2 / v_1}{\sum e_1^2 / v_2} = \frac{\sum e_2^2}{\sum e_1^2} \quad \text{if } v_1 = v_2$$

Step 4:

Calculation:

$$F = \frac{10.68}{1.928} = 5.54$$



Step 5:

Critical Region:

Reject  $H_0$  if

$$F \geq F_{\alpha, v_1, v_2}$$

$$F \geq F_{0.05}(7, 1)$$

$$F \geq 3.787$$

Step 6:

Conclusion:

As  $5.547 > 3.787$ , the calculated value of  $F$  falls in Rejection Region. So, reject  $H_0$  at 5% level of significance i.e. there is heteroscedasticity in the data.

Remedial Measure for heteroscedasticity:

Given that

$$\text{Var}(\epsilon_i) \propto y_i^2$$

$$\text{Var}(\epsilon_i) = \sigma^2 y_i^2$$

$$C_i = \alpha + \beta y_i + \epsilon_i$$

Divide by  $y_i$ ; we get

$$\frac{C_i}{y_i} = \frac{\alpha}{y_i} + \frac{\beta y_i}{y_i} + \frac{\epsilon_i}{y_i}$$

$$w_i = \beta + \alpha v_i + u_i$$

where;

$$w_i = \frac{C_i}{y_i} \quad \epsilon_i \quad v_i = \frac{1}{y_i}$$

$$\hat{\alpha} = \frac{n \sum w_i v_i - (\sum w_i)(\sum v_i)}{n \sum v_i^2 - (\sum v_i)^2} \quad \epsilon_i \quad \beta = \bar{w} - \hat{\alpha} \bar{v}$$

Now, the transformed variables are:

$w_1 = \frac{c_1}{y_1}$	$w_2 = \frac{c_2}{y_2}$	$v_1 = 1/y_1$	$v_2 = 1/y_2$	$\hat{w}_1$	$\hat{w}_2$	$u_1^2$	$u_2^2$
0.8833	0.8450	0.0083	0.005	0.8853	0.8441	0.000041	$3.1 \times 10^{-7}$
0.8852	0.8431	0.0082	0.0049	0.8841	0.8500	$1.21 \times 10^{-6}$	$4.761 \times 10^{-5}$
0.8769	0.8454	0.0077	0.0048	0.8782	0.8560	$1.69 \times 10^{-6}$	$1.123 \times 10^{-4}$
0.8731	0.8565	0.0075	0.0048	0.8758	0.8566	$7.29 \times 10^{-6}$	$2.5 \times 10^{-7}$
0.8786	0.8667	0.0071	0.0048	0.8111	0.8560	$5.62 \times 10^{-5}$	$1.1411 \times 10^{-4}$
0.8667	0.8685	0.0067	0.0047	0.8663	0.8619	$1.6 \times 10^{-7}$	$4.356 \times 10^{-5}$
0.8581	0.8701	0.0065	0.0046	0.8639	0.8679	$3.36 \times 10^{-5}$	$7.84 \times 10^{-6}$
0.8625	0.8682	0.0063	0.0045	0.8616	0.8739	$8.1 \times 10^{-7}$	$3.24 \times 10^{-5}$
						$1.05 \times 10^{-4}$	$3.59 \times 10^{-4}$

$$\hat{w}_1 = 0.787 + 11.84V_1$$

$$\hat{w}_2 = 1.143 - 59.79V_2$$

Null & Alternative Hypothesis:

$H_0$ : There is no heteroscedasticity.

$H_1$ : There is heteroscedasticity.

Level of Significance:

$$\alpha = 0.05$$

Test Statistic:

$$F = \frac{\sum u_2^2}{\sum u_1^2}$$

Calculation:  $F = \frac{3.59 \times 10^{-4}}{1.05 \times 10^{-4}} = 3.42$

Critical Region:

$$F \geq 3.787$$

Conclusion: Accept  $H_0$  i.e. there is no heteroscedasticity.



# PRACTICAL

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## Statement:

Assuming

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

Apply Bartlett's Test for heteroscedasticity by dividing the data into three parts. with No. of workers.

- (i) 100, 150
- (ii) 200, 250
- (iii) 300, 350

X	Y					
100	84	85	86	87	89	90
150	89	91	93	93	94	96
200	95	98	99	103	104	105
250	103	106	109	113	115	117
300	116	118	121	125	127	131
350	136	128	130	132	134	137

## Solution:

### Testing Procedure:

- (1) Null & Alternative hypothesis are:  
 $H_0$ : There is no heteroskedasticity  
 $H_1$ : There is heteroskedasticity

- (2) Level of significance:

$$\alpha = 0.05$$

- (3) Test Statistic:

$$\chi^2 = n \ln \hat{\sigma}_p^2 - \ln v_j \hat{\sigma}_j^2$$
$$1 + \frac{1}{3(m+1)} \left[ \sum \frac{1}{v_j} - \frac{1}{v} \right]$$

- 1) Calculation:

First of all, we have to compute the unbiased estimators of  $\sigma_1^2$ ,  $\sigma_2^2$  &  $\sigma_3^2$ .

we know that

$$\hat{\sigma}_j^2 = \frac{SSE_j}{n - k - 1}$$

and also;

$$\hat{\sigma}_p^2 = \frac{\sum v_j \hat{\sigma}_j^2}{\sum v_j}$$

$$\therefore v_j = n - k - 1$$



1st Group		2nd Group		3rd Group	
$X_1$	$Y_1$	$X_2$	$Y_2$	$X_3$	$Y_3$
100	84	200	95	300	116
100	85	200	98	300	118
100	86	200	99	300	121
100	87	200	103	300	125
100	89	200	104	300	127
100	90	200	105	300	131
150	89	250	103	350	126
150	91	250	106	350	128
150	93	250	109	350	130
150	93	250	113	350	132
150	94	250	115	350	134
150	96	250	117	350	137

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

$$\hat{\beta} = \frac{n \sum XY - (\sum X)(\sum Y)}{n \sum X^2 - (\sum X)^2}$$

$$\hat{\alpha} = \bar{Y} - \hat{\beta} \bar{X}$$

From the above data:

$$(i) \quad \sum X_1 = 1500, \quad \sum Y_1 = 1077, \quad \sum X_1^2 = 195000, \\ \sum Y_1^2 = 96819, \quad \sum X_1 Y_1 = 135500$$

$$\hat{\beta}_1 = 0.117$$

$$\hat{\alpha}_1 = 75.17$$

$$\hat{Y}_1 = 75.17 + 0.117 X_1$$

$$SSE_1 = \sum Y_1^2 - \hat{\alpha}_1 \sum Y_1 - \hat{\beta}_1 \sum X_1 Y_1$$

$$SSE_1 = 96819 - (75.17)(1077) - (0.117)(135500)$$

$$SSE_1 = 7.41$$

$$\hat{\sigma}_1^2 = \frac{SSE_1}{n_1 - k - 1} = \frac{7.41}{12 - 1 - 1} = \frac{7.41}{10} = \boxed{0.741}$$

$$ii) \sum X_2 = 2900, \sum Y_2 = 1267, \sum X_2^2 = 615000, \sum Y_2^2 = 134289$$

$$\sum X_2 Y_2 = 286550$$

$$\hat{\beta}_2 = 0.197, \hat{\alpha}_2 = 61.33$$

$$\hat{Y}_2 = 61.33 + 0.197 X_2$$

$$SSE_2 = \sum Y_2^2 - \hat{\alpha}_2 \sum Y_2 - \hat{\beta}_2 \sum X_2 Y_2$$

$$SSE_2 = 134289 - (61.33)(1267) - (0.197)(286550)$$

$$SSE_2 = 133.54$$

$$\hat{\sigma}_2^2 = \frac{SSE_2}{n_2 - k - 1} = \frac{133.54}{12 - 1 - 1} = \boxed{13.354}$$

$$i) \sum X_3 = 3900, \sum Y_3 = 1535, \sum X_3^2 = 1275000$$

$$\sum Y_3^2 = 196865, \sum X_3 Y_3 = 500350$$

$$\hat{\beta}_3 = 0.1633, \hat{\alpha}_3 = 74.00$$

$$\hat{Y}_3 = 74.0 + 0.1633 X_3$$

$$SSE_3 = \sum Y_3^2 - \hat{\alpha}_3 \sum Y_3 - \hat{\beta}_3 \sum X_3 Y_3$$

$$SSE_3 = 194245 - (74.0)(1525) - (0.1633)(496850)$$

$$SSE_3 = 259.395$$

$$\hat{\sigma}_3^2 = \frac{SSE_3}{n_3 - k - 1} = \frac{259.395}{12 - 1 - 1} = \boxed{25.9395}$$



$n_j$	$x_j = d_j$	$\hat{p}_j$	$n_j \hat{p}_j$	$n_j \ln \hat{p}_j$	$4 n_j$
12	10	0.711	7.41	-2.99	0.10
12	10	13.381	133.81	25.918	0.10
12	10	25.935	259.395	32.56	0.10
			400.345	55.418	0.30

$$\hat{p}_j = \frac{\sum n_j \hat{p}_j}{\sum n_j} = \frac{400.345}{30} = 13.345$$

$$\chi^2 = (30) \ln(13.345) = 55.48$$

$$1 + \frac{1}{3(3+1)} \left( 0.30 - \frac{1}{30} \right)$$

$$\chi^2 = 5.89$$

(5)

Critical Region:

Reject  $H_0$  if

$$\chi^2 \geq \chi_{\alpha, m-1}^2$$

$$\chi^2 \geq \chi_{0.05, 2}^2$$

$$\chi^2 \geq 5.99$$

(6)

Conclusion:

As  $5.89 < 5.99$ , accept  $H_0$  at 5% level of significance i.e. there is no heterogeneity in the data.

# PRACTICAL # 12

## Statement:

The following table gives the level of import (Y) and GDP (X), both seasonally adjusted in 100 billions from 1980 to 1991. Regress Y on X and test for autocorrelation by D.W. test. Estimate first order autocorrelation by D.W. statistic and correct for autocorrelation if any.

$Y_t$ :	29.92	31.94	29.49	35.80	41.64	43.89	46.77
	53.67	57.35	59.96	64.92	63.90	68.71	74.49
	85.96	90.93	99.28	108.70	114.73	133.01	

$X_t$ :	29.19	32.08	33.16	36.89	40.33	43.19	45.37
	48.92	52.58	55.88	58.47	60.81	64.70	67.95
	72.18	72.29	79.81	84.79	89.75	95.60	



Solution:

$Y_t$	$X_t$	$Y_t$	$X_t$
29.92	29.19	64.92	58.47
31.94	32.03	63.90	60.81
29.49	33.16	68.71	64.70
35.80	36.89	74.49	67.95
41.64	40.33	85.96	72.18
43.89	43.19	90.93	72.29
46.77	45.37	99.28	79.81
53.67	48.92	108.70	84.79
57.35	58.58	114.73	89.75
59.96	55.88	133.01	95.60

The Model is:

$$Y_t = \alpha + \beta X_t + \epsilon_t$$

$$\sum X_t = 1163.89, \quad \sum X_t^2 = 75278.2425, \quad \sum Y_t = 1335.06$$

$$\sum Y_t^2 = 106430.8862, \quad \sum X_t Y_t = 89003.0925$$

$$\begin{pmatrix} X'X \\ n \end{pmatrix} = \begin{bmatrix} n & \sum X_t \\ \sum X_t & \sum X_t^2 \end{bmatrix} = \begin{bmatrix} 20 & 1163.89 \\ 1163.89 & 75278.2425 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} \sum Y_t \\ \sum X_t Y_t \end{bmatrix} = \begin{bmatrix} 1335.06 \\ 89003.0925 \end{bmatrix}$$

$$(X'X)^{-1} = \begin{bmatrix} 0.4988 & -0.0077117 \\ -0.0077117 & 0.00013252 \end{bmatrix}$$

$$\underline{\beta} = (X'X)^{-1} X'Y$$

$$\underline{\beta} = \begin{bmatrix} 0.4988 & -0.0077117 \\ -0.0077117 & 0.00013252 \end{bmatrix} \begin{bmatrix} 1335.06 \\ 89003.0925 \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} -20.4661 \\ 1.4988 \end{bmatrix}$$

hence;

$$Y_t = -20.4661 + 1.4988 X_t$$

Testing Procedure:

1) Null & Alternative hypothesis are:

H<sub>0</sub>: There is no autocorrelation i.e.  $\rho = 0$ .

H<sub>1</sub>: There is autocorrelation i.e.  $\rho \neq 0$ .

2) Level of significance:

$$\alpha = 0.05$$

Test Statistic:

$$d = \frac{\sum_{t=1}^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2}$$

$$d = 2 - 2\hat{\rho} = 2(1 - \hat{\rho})$$

$$\Rightarrow \hat{\rho} = \frac{2-d}{2}$$

$$\hat{\rho} = 1 - \frac{d}{2}$$



(4) Calculation:

$X_t$	$Y_t$	$\hat{Y}_t = -20.46 + 1.49X_t$	$e_t$	$e_{t-1}$	$(e_t - e_{t-1})^2$
29.19	29.92	23.28	6.63	—	—
32.03	31.94	27.50	4.39	6.63	5.0024
33.16	29.49	29.23	0.25	4.39	17.1694
36.89	35.80	34.82	0.97	0.25	0.5171
40.33	41.64	39.98	1.65	0.97	0.4679
43.19	43.89	44.26	-2.62	1.65	0.9281
45.37	46.77	47.53	-0.76	-2.62	11.4745
48.92	53.67	52.85	0.81	-0.76	2.4942
52.58	57.35	58.34	-0.99	0.81	3.2602
55.88	59.96	63.28	-3.32	-0.99	5.4569
58.47	64.92	67.16	-2.24	-3.32	10.1623
60.81	63.90	70.67	-6.77	-2.24	20.4955
64.70	68.71	76.50	-7.79	-6.77	1.0412
76.95	74.49	81.37	-6.88	-7.79	0.8261
72.18	85.96	87.71	-1.75	-6.88	26.3179
72.29	90.93	87.88	3.04	-1.75	23.0889
79.81	99.28	99.15	0.12	3.04	8.5317
84.79	108.70	106.61	2.08	0.12	3.8255
89.75	114.73	114.05	0.67	2.08	1.9712
95.6	133.01	122.81	10.19	0.67	90.4781
			$\sum e_t = 0$		
			$\sum e_t^2 = 367.5741$		221.5097

$$\sum e_t^2 = 367.5741$$

$$\Rightarrow d = \frac{224.5097}{367.5741} = 0.6107$$

(5) Critical Region:

$$k=1, \quad n=20, \quad \alpha=0.05$$

$$d_L = 1.201, \quad d_U = 1.411$$

Critical Region	No Conclusion	Acceptance Region
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$d_i$	0	1.201	1.411	2	$d_U$
$e_i$	1	$d_L$	$d_U$	0	-1

(6) Conclusions

since  $0.6107 < d_L$ , so reject  $H_0$   
at 5% level of significance i.e. there  
is positive autocorrelation in the data.

Now; for the correction:

As;

$$Y_t = \alpha + \beta X_t + \epsilon_t \quad \text{--- (i)}$$

$$Y_{t-1} = \alpha + \beta X_{t-1} + \epsilon_{t-1} \quad \text{--- (ii)}$$

$$\hat{e}Y_{t-1} = \hat{e}\alpha + \hat{e}\beta X_{t-1} + \hat{e}\epsilon_{t-1} \quad \text{--- (iii)}$$

$$\text{eq. (i)} - \text{eq. (iii)}$$

$$Y_t - \hat{e}Y_{t-1} = \alpha(1 - \hat{e}) + \beta(X_t - \hat{e}X_{t-1}) + (\epsilon_t - \hat{e}\epsilon_{t-1})$$

$$Y_{*t} = \alpha_0 + \beta X_{*t} + u_t$$



$$as; \quad \hat{e} = \frac{1-d}{2} = \frac{1-0.6107}{2} = 0.695$$

$Y_t$	$Y_{t-1}$	$Y_{xt} = Y_t - \hat{e}Y_{t-1}$	$X_t$	$X_{t-1}$	$X_{xt} = X_t - \hat{e}X_{t-1}$
29.92	—	—	29.19	—	—
31.94	29.92	11.1456	32.03	29.19	11.7429
29.49	31.94	7.2917	33.16	32.03	10.8992
35.80	29.49	15.3015	36.89	33.16	13.8438
41.64	35.80	16.7590	40.33	36.89	14.6915
43.89	41.64	14.9502	43.19	40.33	15.1607
46.77	43.89	16.2665	45.31	43.19	15.3529
53.67	46.77	21.1649	48.92	45.37	17.3879
57.35	53.67	20.0494	52.58	48.92	18.5806
59.96	57.35	20.1018	55.88	52.58	19.3369
64.92	59.96	23.2478	58.47	55.88	19.6334
63.90	64.92	18.7866	60.81	58.47	20.1734
68.71	63.90	24.2995	64.70	60.81	22.4371
74.49	68.71	26.7366	67.95	64.70	22.9835
85.96	74.49	34.1895	72.18	67.95	24.9848
90.93	85.96	31.1878	72.29	72.18	22.1249
99.28	90.93	36.0837	79.81	72.29	29.5685
108.70	99.28	39.7004	84.79	79.81	29.3221
114.73	108.70	39.1835	89.75	84.79	30.8209
133.01	114.73	53.2727	95.60	89.75	33.2238
		436.7157			392.2388

$$\sum X_{xt} Y_{xt} = 11020.94984$$

$$\sum X_{xt}^2 = 8875.415$$

$$\bar{Y}_{xt} = 24.7218$$

$$\bar{X}_{xt} = 20.644$$

$$\hat{\beta} = \frac{\sum X_{*t} Y_{*t} - (\sum X_{*t})(\sum Y_{*t})/n}{\sum X_{*t}^2 - (\sum X_{*t})^2/n}$$

$$\hat{\beta} = \frac{11020.94984 - (392.23)(436.7157)/19}{8875.415 - (392.23)^2/19}$$

$$\hat{\beta} = 1.702$$

and,

$$\hat{\alpha}_0 = \bar{Y}_{*t} - \hat{\beta} \bar{X}_{*t}$$

$$\hat{\alpha}_0 = 24.7218 - (1.702)(20.814)$$

$$\hat{\alpha}_0 = -10.41305$$

Now,

$$\alpha = \frac{\hat{\alpha}_0}{1 - \hat{\beta}} = \frac{-10.41305}{1 - 0.695}$$

$$\Rightarrow \alpha = -34.141$$

Hence, the estimated transformed model is:

$$Y_{*t} = -34.141 + 1.702 X_{*t}$$



# PRACTICAL # 13

## Statement:

A company which manufactures automotive parts, wishes to model the effect of advertising on sales. The advertising expenditure each month ( $X$ ) and the sales volume ( $Y$ ) each month for the last two years are given as below.

Month	$Y_t$	$X_t$	Month	$Y_t$	$X_t$
1	92.8	25	13	85.4	15
2	79.2	0	14	80.4	5
3	84.5	15	15	83.5	10
4	83.0	10	16	92.5	25
5	88.1	20	17	89.5	15
6	83.9	10	18	83.6	5
7	79.7	5	19	89.1	15
8	81.1	5	20	90.9	20
9	86.4	15	21	92.7	25
10	86.3	15	22	88.1	15
11	79.9	5	23	79.5	0
12	86.6	20	24	82.9	5

Estimate the model;  $Y_t = \beta_0 + \beta_1 X_t + \beta_2 t + \epsilon_t$  where  $T = \text{Month}$ . Apply Geary test to check the presence of autocorrelation.

Estimate the autocorrelation co-efficient by the two step procedure. Cochran arcsin

## Solution:

The estimated model is:

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 X_t + \hat{\beta}_2 t_2 + \epsilon_t$$

From the data;

$$\begin{aligned} \sum Y_t &= 2049.8, & \sum Y_t^2 &= 175506.20, & \sum X_t &= 300.00 \\ \sum X_t^2 &= 5100, & \sum X_t Y_t &= 26344.5, & \sum t_2 &= 300.0 \\ \sum t_2^2 &= 4900, & \sum X_t t_2 &= 3745, & \sum Y_t t_2 &= 25755.90 \end{aligned}$$

$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$X'X = \begin{bmatrix} n & \sum X_t & \sum t_2 \\ \sum X_t & \sum X_t^2 & \sum X_t t_2 \\ \sum t_2 & \sum X_t t_2 & \sum t_2^2 \end{bmatrix} = \begin{bmatrix} 24 & 300 & 300 \\ 300 & 5100 & 3745 \\ 300 & 3745 & 4900 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} \sum Y_t \\ \sum X_t Y_t \\ \sum t_2 Y_t \end{bmatrix} = \begin{bmatrix} 2049.8 \\ 26344.5 \\ 25755.9 \end{bmatrix} \quad \text{--- (1)}$$

$$(X'X)^{-1} = \begin{bmatrix} 0.29429 & -0.009299 & -0.01090 \\ -0.009299 & 0.00074075 & 3.22 \times 10^{-6} \\ -0.01090 & 3.22 \times 10^{-6} & 0.0008695 \end{bmatrix} \quad \text{--- (2)}$$

hence;

$$\hat{\beta} = \begin{bmatrix} 77.2386 \\ 0.5353 \\ 0.1183 \end{bmatrix}$$

by (1) & (2)



The estimated model is:

$$Y_t = 77.2386 + 0.5353X_t + 0.1183t_2$$

### Testing Procedure:

- (1) Null & Alternative hypothesis are:  
H<sub>0</sub>: There is no autocorrelation i.e.  $\rho = 0$   
H<sub>1</sub>: There is autocorrelation i.e.  $\rho \neq 0$

- (2) Level of significance:  
 $\alpha = 0.05$

- (3) Test Statistic:  
$$Z = \frac{R - \text{Mean}(R)}{S.E.(R)}$$

$$\text{Mean}(R) = \frac{2N_1N_2}{N} + 1$$

$$S.E.(R) = \frac{2N_1N_2(2N_1N_2 - N)}{N^2(N-1)}$$

### Calculation:

$N_1$  = Number of positive residuals

$N_2$  = Number of negative residuals

$R$  = No. of Runs

$$N = N_1 + N_2$$

$Y_t$	$X_t$	$t_2$	$\hat{Y}_t = 77.23 + 0.535X_t + 0.118t_2$	$e_t = Y_t - \hat{Y}_t$
92.8	25	1	90.73	2.06
79.2	0	2	77.47	1.72
84.5	15	3	85.62	-1.12
83.0	10	4	83.04	-0.06
88.1	20	5	88.53	-0.43
83.9	10	6	83.30	0.59
79.9	5	7	80.74	-0.84
81.1	5	8	80.86	0.23
86.4	15	9	86.33	0.06
86.3	15	10	86.45	-0.15
79.9	5	11	81.21	-1.31
86.6	20	12	89.36	-2.76
85.4	15	13	86.80	-1.40
80.4	5	14	81.57	-1.17
83.5	10	15	84.36	-0.86
92.5	25	16	92.51	-0.013
89.5	15	17	87.27	2.22
83.6	5	18	82.04	1.55
89.1	15	19	87.51	1.58
90.9	20	20	90.31	0.58
92.7	25	21	93.10	-0.40
88.1	15	22	87.87	0.22
79.5	0	23	79.95	-0.45
82.9	5	24	82.75	0.14



here,  $N_1 = 11$ ,  $N_2 = 13$ ,  $N = N_1 + N_2 = 24$

$$\text{Mean}(R) = \frac{2(11)(13)}{24} + 1 = 12.9167$$

$$S.E.(R) = \frac{2(11)(13) \{2(11)(13) - 24\}}{(24)^2(24-1)} = 5.6561$$

$$R = 11, \quad S.E.(R) = 2.378$$

So;

$$Z = \frac{11 - 12.9167}{2.378} = -0.816$$

(5) Critical Region:

Reject  $H_0$  if

$$|Z| \geq Z_{\alpha/2}$$

$$|Z| \geq Z_{0.025}$$

$$|Z| \geq 1.96$$

(6) Conclusion:

Since  $Z_{cal} = -0.816$  falls in acceptance Region so accept  $H_0$  at 5% level of significance i.e. there is no autocorrelation in the data.

Now, to estimate the correlation co-efficient by the Cochran-Cox's Two step procedure

$$\hat{\rho} = \frac{\sum e_t e_{t-1}}{\sum e_t^2}$$

$e_t$	$e_{t-1}$	$e_t e_{t-1}$	$e_t$	$e_{t-1}$	$e_t e_{t-1}$
2.06	—	—	-1.40	-2.76	3.88
1.72	2.06	3.55	-1.17	-1.40	1.64
-0.12	1.72	-1.93	-0.86	-1.17	1.01
-0.06	-1.12	0.07	-0.013	-0.26	0.01
-0.43	-0.06	0.02	2.22	-0.013	-0.03
0.59	-0.43	-0.25	1.55	2.22	3.45
-0.84	0.59	-0.50	1.58	1.55	2.46
0.23	-0.84	-0.20	0.58	1.58	0.93
0.06	0.23	0.01	-0.40	0.58	-0.23
-0.15	0.06	-0.01	0.22	-0.40	-0.09
-1.31	-0.15	0.19	-0.45	0.22	-0.10
-2.76	-1.31	3.63	0.41	-0.45	-0.06

$$\sum e_t e_{t-1} = 17.4754$$

$$\sum e_t^2 = 33.9686$$

so,

$$\hat{\rho} = \frac{17.4754}{33.9689}$$

$$\hat{\rho} = 0.514$$